



# UNIVERSITY OF CAMBRIDGE

## Department of Chemistry

### Data Book (revised October 2005)

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<b>H</b> 1 1.008																	<b>He</b> 2 4.003
<b>Li</b> 3 6.94	<b>Be</b> 4 9.01	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <b>symbol</b> atomic number mean atomic mass         </div>										<b>B</b> 5 10.81	<b>C</b> 6 12.01	<b>N</b> 7 14.01	<b>O</b> 8 16.00	<b>F</b> 9 19.00	<b>Ne</b> 10 20.18
<b>Na</b> 11 22.99	<b>Mg</b> 12 24.31											<b>Al</b> 13 26.98	<b>Si</b> 14 28.09	<b>P</b> 15 30.97	<b>S</b> 16 32.06	<b>Cl</b> 17 35.45	<b>Ar</b> 18 39.95
<b>K</b> 19 39.102	<b>Ca</b> 20 40.08	<b>Sc</b> 21 44.96	<b>Ti</b> 22 47.90	<b>V</b> 23 50.94	<b>Cr</b> 24 52.00	<b>Mn</b> 25 54.94	<b>Fe</b> 26 55.85	<b>Co</b> 27 58.93	<b>Ni</b> 28 58.71	<b>Cu</b> 29 63.55	<b>Zn</b> 30 65.37	<b>Ga</b> 31 69.72	<b>Ge</b> 32 72.59	<b>As</b> 33 74.92	<b>Se</b> 34 78.96	<b>Br</b> 35 79.904	<b>Kr</b> 36 83.80
<b>Rb</b> 37 85.47	<b>Sr</b> 38 87.62	<b>Y</b> 39 88.91	<b>Zr</b> 40 91.22	<b>Nb</b> 41 92.91	<b>Mo</b> 42 95.94	<b>Tc</b> 43	<b>Ru</b> 44 101.07	<b>Rh</b> 45 102.91	<b>Pd</b> 46 106.4	<b>Ag</b> 47 107.87	<b>Cd</b> 48 112.40	<b>In</b> 49 114.82	<b>Sn</b> 50 118.69	<b>Sb</b> 51 121.75	<b>Te</b> 52 127.60	<b>I</b> 53 126.90	<b>Xe</b> 54 131.30
<b>Cs</b> 55 132.91	<b>Ba</b> 56 137.34	<b>La*</b> 57 138.91	<b>Hf</b> 72 178.49	<b>Ta</b> 73 180.95	<b>W</b> 74 183.85	<b>Re</b> 75 186.2	<b>Os</b> 76 190.2	<b>Ir</b> 77 192.2	<b>Pt</b> 78 195.09	<b>Au</b> 79 196.97	<b>Hg</b> 80 200.59	<b>Tl</b> 81 204.37	<b>Pb</b> 82 207.2	<b>Bi</b> 83 208.98	<b>Po</b> 84	<b>At</b> 85	<b>Rn</b> 86
<b>Fr</b> 87	<b>Ra</b> 88	<b>Ac<sup>+</sup></b> 89															

<b>*Lanthanides</b>	<b>Ce</b> 58 140.12	<b>Pr</b> 59 140.91	<b>Nd</b> 60 144.24	<b>Pm</b> 61	<b>Sm</b> 62 150.4	<b>Eu</b> 63 151.96	<b>Gd</b> 64 157.25	<b>Tb</b> 65 158.93	<b>Dy</b> 66 162.50	<b>Ho</b> 67 164.93	<b>Er</b> 68 167.26	<b>Tm</b> 69 168.93	<b>Yb</b> 70 173.04	<b>Lu</b> 71 174.97
<b>+Actinides</b>	<b>Th</b> 90 232.01	<b>Pa</b> 91	<b>U</b> 92 238.03	<b>Np</b> 93	<b>Pu</b> 94	<b>Am</b> 95	<b>Cm</b> 96	<b>Bk</b> 97	<b>Cf</b> 98	<b>Es</b> 99	<b>Fm</b> 100	<b>Md</b> 101	<b>No</b> 102	<b>Lr</b> 103

## Constants

Name	Symbol and definition	Value (uncertainty)	Unit
	$\pi$	3.141592653589 ...	
	$e$	2.718281828459 ...	
	$\ln 10 = 1/\log_{10} e$	2.302585092994 ...	
Speed of light	$c$	2.99792458	$10^8 \text{ m s}^{-1}$
Planck's constant	$h$	6.6260693(11)	$10^{-34} \text{ J s}$
	$\hbar = h/2\pi$	1.05457168(18)	$10^{-34} \text{ J s}$
Avogadro's constant	$N_A$	6.0221415(10)	$10^{23} \text{ mol}^{-1}$
Elementary charge	$e$	1.60217653(14)	$10^{-19} \text{ C}$
Electron rest mass	$m_e$	0.91093826(16)	$10^{-30} \text{ kg}$
Atomic mass unit	$m_u = 1 \text{ g mol}^{-1} / N_A$	1.66053886(28)	$10^{-27} \text{ kg}$
Proton rest mass	$m_p$	1.67262171(29)	$10^{-27} \text{ kg}$
Neutron rest mass	$m_n$	1.67492728(29)	$10^{-27} \text{ kg}$
Faraday constant	$F = N_A e$	9.64853383(83)	$10^4 \text{ C mol}^{-1}$
Boltzmann constant	$k_B$	1.3806505(24)	$10^{-23} \text{ J K}^{-1}$
Molar Gas constant	$R = N_A k_B$	8.314472(15)	$\text{J mol}^{-1} \text{ K}^{-1}$
Permeability of vacuum	$\mu_0$	$4\pi \times 10^{-7}$	$\text{H m}^{-1}$
Permittivity of vacuum	$\epsilon_0 = 1/(\mu_0 c^2)$	8.8541878 ...	$10^{-12} \text{ F m}^{-1}$
	$4\pi\epsilon_0$	1.1126501 ...	$10^{-10} \text{ F m}^{-1}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	9.27400949(80)	$10^{-24} \text{ J T}^{-1}$
Nuclear magneton	$\mu_N = e\hbar/2m_p$	5.05078343(43)	$10^{-27} \text{ J T}^{-1}$
Stefan-Boltzmann constant	$\sigma = 2\pi^5 k_B^4 / 15h^3 c^2$	5.670400(40)	$10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Bohr radius	$a_0 = 4\pi\epsilon_0 \hbar^2 / m_e e^2$	0.5291772108(18)	$10^{-10} \text{ m}$
Hartree energy	$E_h = e^2 / 4\pi\epsilon_0 a_0$	4.35974417(75)	$10^{-18} \text{ J}$
Fine structure constant	$\alpha = e^2 / 4\pi\epsilon_0 \hbar c$	7.297352568(24)	$10^{-3}$
	$\alpha^{-1}$	137.035986	

CODATA recommended values, December 2002

<http://physics.nist.gov/constants>

The estimated standard uncertainty, in parentheses after the value, applies to the least significant digits of the value.

## Energy Conversion Factors

	J	kJ mol <sup>-1</sup>	cm <sup>-1</sup>	K
1 J	1	6.0221 · 10 <sup>20</sup>	5.0341 · 10 <sup>22</sup>	7.2429 · 10 <sup>22</sup>
1 hartree	4.35974 · 10 <sup>-18</sup>	2625.5	219475	315773
1 eV	1.60218 · 10 <sup>-19</sup>	96.485	8065.54	11604
1 kJ mol <sup>-1</sup>	1.66054 · 10 <sup>-21</sup>	1	83.5935	120.27
1 cm <sup>-1</sup>	1.98645 · 10 <sup>-23</sup>	11.963 · 10 <sup>-3</sup>	1	1.4388
1 K	1.38065 · 10 <sup>-23</sup>	8.3145 · 10 <sup>-3</sup>	0.69504	1
1 Hz	6.62607 · 10 <sup>-34</sup>	3.9903 · 10 <sup>-13</sup>	3.3356 · 10 <sup>-11</sup>	4.7992 · 10 <sup>-11</sup>

Note: the energy of a photon with reciprocal wavelength (wavenumber)  $1/\lambda$  and frequency  $\nu$  is  $hc/\lambda = h\nu$ . The energy corresponding to a temperature  $T$  is  $k_B T$ .

## Other conversion factors

Length	Å	$10^{-10} \text{ m}$
Energy	cal	4.184 J
Pressure	atm = 760 Torr	101325 Pa
	Torr = mm Hg	133.3 Pa
	bar	$10^5 \text{ Pa}$
Radioactivity	becquerel, Bq	$1 \text{ s}^{-1}$
	curie, Ci	$3.7 \cdot 10^{10} \text{ Bq}$
Charge	esu	$3.33564 \cdot 10^{-10} \text{ C}$
Dipole moment	debye = $10^{-18} \text{ esu cm}$	$3.33564 \cdot 10^{-30} \text{ C m}$
	a.u. = $e a_0$	$8.478358 \cdot 10^{-30} \text{ C m}$
Temperature	°C	$0^\circ \text{C} = 273.15 \text{ K}$

The 'entropy unit' (e.u.) used for entropies of activation is usually the c.g.s. unit cal/mol/°C. However some authors use the same symbol for the SI unit,  $\text{J mol}^{-1} \text{ K}^{-1}$ .

## Greek Alphabet

A	$\alpha$	alpha	H	$\eta$	eta	N	$\nu$	nu	T	$\tau$	tau
B	$\beta$	beta	$\Theta$	$\theta, \vartheta$	theta	$\Xi$	$\xi$	xi	$\Upsilon$	$\upsilon$	upsilon
$\Gamma$	$\gamma$	gamma	I	$\iota$	iota	O	$o$	omicron	$\Phi$	$\phi, \varphi$	phi
$\Delta$	$\delta$	delta	K	$\kappa$	kappa	$\Pi$	$\pi$	pi	X	$\chi$	chi
E	$\varepsilon$	epsilon	$\Lambda$	$\lambda$	lambda	P	$\rho$	rho	$\Psi$	$\psi$	psi
Z	$\zeta$	zeta	M	$\mu$	mu	$\Sigma$	$\sigma$	sigma	$\Omega$	$\omega$	omega

## Series

### Geometrical progression

$$S_n = a + az + az^2 + \dots + az^{n-1} = a \frac{1 - z^n}{1 - z}. \quad S_\infty = \frac{a}{1 - z} \quad \text{when } |z| < 1.$$

### Power series

$$\exp z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots + \frac{z^n}{n!} + \dots$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \dots$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \dots$$

$$(1 + z)^p = 1 + pz + \frac{p(p-1)}{2!}z^2 + \frac{p(p-1)(p-2)}{3!}z^3 + \dots, \quad |z| < 1$$

$$\ln(1 + z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \dots - \frac{(-z)^n}{n} - \dots, \quad |z| < 1$$

## Stirling's formula

$$\ln n! \approx n \ln n - n \quad \text{for large } n$$

## Determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

In general,

$$\det(\mathbf{A}) = \sum_j A_{ij} C_{ji} \quad (i \text{ fixed at any value}),$$

where the cofactor  $C_{ji}$  is  $(-1)^{i+j}$  times the determinant of the matrix obtained by deleting the  $i$ th row and the  $j$ th column of  $\mathbf{A}$ . For example,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} + a_{12}a_{23}a_{31} - a_{12}a_{21}a_{33} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}.$$

## Integrals

$$\int_{-\infty}^{\infty} \exp(-ax^2) dx = \sqrt{\pi/a}, \quad (a > 0)$$

$$\int_{-\infty}^{\infty} x^{2n} \exp(-ax^2) dx = 1 \times 3 \times 5 \times \dots \times (2n-1) \frac{\sqrt{\pi/a}}{(2a)^n} \quad (n \geq 1; a > 0)$$

$$\int_0^{\infty} r^n \exp(-ar) dr = \frac{n!}{a^{n+1}} \quad (n \geq 0; a > 0)$$

$$\begin{aligned} \int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta &= \frac{m-1}{m+n} \int_0^{\pi/2} \sin^{m-2} \theta \cos^n \theta d\theta \\ &= \frac{n-1}{m+n} \int_0^{\pi/2} \sin^m \theta \cos^{n-2} \theta d\theta, \end{aligned}$$

so that

$$\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = \frac{(m-1)(m-3)\dots(n-1)(n-3)\dots}{(m+n)(m+n-2)(m+n-4)\dots} \times C,$$

where  $C = \pi/2$  if  $m$  and  $n$  are both *even*, and  $C = 1$  otherwise. E.g.:

$$\int_0^{\pi/2} \sin \theta \cos^3 \theta d\theta = \frac{2}{4.2} = \frac{1}{4}; \quad \int_0^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta = \frac{1.1 \pi}{4.2 \cdot 2} = \frac{\pi}{16}.$$

*Integration by parts*

$$\int_a^b u \frac{dv}{dx} dx = [uv]_a^b - \int_a^b \frac{du}{dx} v dx$$

## Trigonometrical formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos A \cos B = \frac{1}{2} (\cos(A+B) + \cos(A-B))$$

$$\sin A \sin B = \frac{1}{2} (\cos(A-B) - \cos(A+B))$$

$$\sin A \cos B = \frac{1}{2} (\sin(A+B) + \sin(A-B))$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

*Cosine formula*

$$a^2 = b^2 + c^2 - 2bc \cos A$$

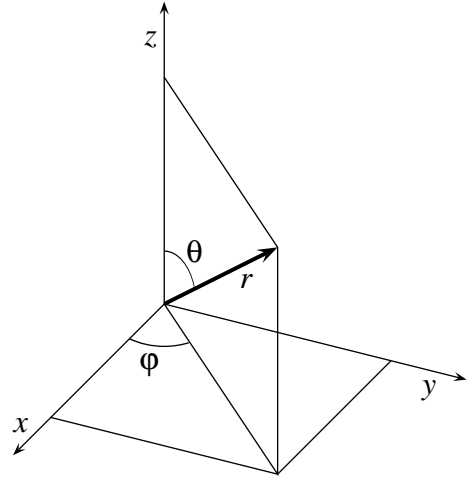
## Spherical Polar Coordinates

*Relationship with Cartesian coordinates*

$$\begin{aligned} x &= r \sin \theta \cos \varphi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \varphi & \theta &= \arccos(z/r) \\ z &= r \cos \theta & \varphi &= \arctan(y/x) \end{aligned}$$

*Integration*

$$\int \dots dV = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \dots r^2 \sin \theta dr d\theta d\varphi$$



*Laplacian*

$$\begin{aligned} \nabla^2 \psi &= \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \varphi^2} \end{aligned}$$

*Spherical Harmonics*

$$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}} C_{lm}(\theta, \varphi),$$

where

$$C_{lm}(\theta, \varphi) = \left[ \frac{(l-|m|)!}{(l+|m|)!} \right]^{\frac{1}{2}} P_l^{|m|}(\cos \theta) e^{im\varphi} \times \begin{cases} (-1)^m & \text{for } m > 0, \\ 1 & \text{for } m \leq 0. \end{cases}$$

Here  $P_l^m$  is an associated Legendre polynomial. In particular,

$$C_{00} = 1,$$

$$C_{10} = \cos \theta = z/r,$$

$$C_{1,\pm 1} = \mp \sqrt{\frac{1}{2}} \sin \theta e^{\pm i\varphi} = \mp \sqrt{\frac{1}{2}} (x \pm iy)/r,$$

$$C_{20} = \frac{1}{2} (3 \cos^2 \theta - 1) = \frac{1}{2} (3z^2 - r^2)/r^2,$$

$$C_{2,\pm 1} = \mp \sqrt{\frac{3}{2}} \cos \theta \sin \theta e^{\pm i\varphi} = \mp \sqrt{\frac{3}{2}} (zx \pm izy)/r^2,$$

$$C_{2,\pm 2} = \sqrt{\frac{3}{8}} \sin^2 \theta e^{\pm 2i\varphi} = \sqrt{\frac{3}{8}} (x^2 - y^2 \pm 2ixy)/r^2.$$

*Ladder Operators*

$$\hat{J}_{\pm} \equiv \hat{J}_x \pm i\hat{J}_y; \quad \hat{J}_{\pm} |J, M\rangle = \sqrt{J(J+1) - M(M \pm 1)} |J, M \pm 1\rangle$$

## Character tables for some important symmetry groups

$C_i$	$E$	$i$	
$A_g$	1	1	$R_x; R_y; R_z \quad x^2; y^2; z^2; xy; xz; yz$
$A_u$	1	-1	$x; y; z$

$C_s$	$E$	$\sigma_h$	
$A'$	1	1	$x; y \quad R_z \quad x^2; y^2; z^2; xy$
$A''$	1	-1	$z \quad R_x; R_y \quad xz; yz$

$C_2$	$E$	$C_2^z$	
$A$	1	1	$z \quad R_z \quad x^2; y^2; z^2; xy$
$B$	1	-1	$x; y \quad R_x; R_y \quad xz; yz$

$C_{2v}$	$E$	$C_2^z$	$\sigma^{xz}$	$\sigma^{yz}$	
$A_1$	1	1	1	1	$z \quad x^2; y^2; z^2$
$A_2$	1	1	-1	-1	$R_z \quad xy$
$B_1$	1	-1	1	-1	$x \quad R_y \quad xz$
$B_2$	1	-1	-1	1	$y \quad R_x \quad yz$

$C_{2h}$	$E$	$C_2^z$	$i$	$\sigma^{xy}$	
$A_g$	1	1	1	1	$R_z \quad x^2; y^2; z^2; xy$
$B_g$	1	-1	1	-1	$R_x; R_y \quad xz; yz$
$A_u$	1	1	-1	-1	$z$
$B_u$	1	-1	-1	1	$x; y$

$D_2$	$E$	$C_2^z$	$C_2^y$	$C_2^x$	
$A$	1	1	1	1	$x^2; y^2; z^2$
$B_1$	1	1	-1	-1	$z \quad R_z \quad xy$
$B_2$	1	-1	1	-1	$y \quad R_y \quad xz$
$B_3$	1	-1	-1	1	$x \quad R_x \quad yz$

$D_{2d}$	$E$	$2S_4$	$C_2^z$	$2C_2'$	$2\sigma_d$	
$A_1$	1	1	1	1	1	$x^2 + y^2; z^2$
$A_2$	1	1	1	-1	-1	$R_z$
$B_1$	1	-1	1	1	-1	$x^2 - y^2$
$B_2$	1	-1	1	-1	1	$z \quad xy$
$E$	2	0	-2	0	0	$(x, y) \quad (R_x, R_y) \quad (xz, yz)$

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$\mathcal{D}_{2h}$	$E$	$C_2^z$	$C_2^y$	$C_2^x$	$i$	$\sigma^{xy}$	$\sigma^{xz}$	$\sigma^{yz}$	
$A_g$	1	1	1	1	1	1	1	1	$x^2; y^2; z^2$
$B_{1g}$	1	1	-1	-1	1	1	-1	-1	$R_z$ $xy$
$B_{2g}$	1	-1	1	-1	1	-1	1	-1	$R_y$ $xz$
$B_{3g}$	1	-1	-1	1	1	-1	-1	1	$R_x$ $yz$
$A_u$	1	1	1	1	-1	-1	-1	-1	
$B_{1u}$	1	1	-1	-1	-1	-1	1	1	$z$
$B_{2u}$	1	-1	1	-1	-1	1	-1	1	$y$
$B_{3u}$	1	-1	-1	1	-1	1	1	-1	$x$

$C_3$	$E$	$C_3$	$C_3^2$	$\omega = \exp(2\pi i/3)$			
$A$	1	1	1	$z$	$R_z$	$x^2 + y^2; z^2$	
$E$ {	1	$\omega$	$\omega^2$	$x - iy$	$R_x - iR_y$	$xz - iyz; x^2 + 2ixy - y^2$	
	1	$\omega^2$	$\omega$	$x + iy$	$R_x + iR_y$	$xz + iyz; x^2 - 2ixy - y^2$	

$C_{3v}$	$E$	$2C_3$	$3C_2$			
$A_1$	1	1	1	$z$		$x^2 + y^2; z^2$
$A_2$	1	1	-1		$R_z$	
$E$	2	-1	0	$(x, y)$	$(R_x, R_y)$	$(xz, yz); (x^2 - y^2, 2xy)$

$\mathcal{D}_3$	$E$	$2C_3$	$3C_2$			
$A_1$	1	1	1			$x^2 + y^2; z^2$
$A_2$	1	1	-1	$z$	$R_z$	
$E$	2	-1	0	$(x, y)$	$(R_x, R_y)$	$(xz, yz); (x^2 - y^2, 2xy)$

$\mathcal{D}_{3d}$	$E$	$2C_3$	$3C_2$	$i$	$2S_6$	$3\sigma_d$	
$A_{1g}$	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_{2g}$	1	1	-1	1	1	-1	$R_z$
$E_g$	2	-1	0	2	-1	0	$(R_x, R_y)$ $(xz, yz); (x^2 - y^2, 2xy)$
$A_{1u}$	1	1	1	-1	-1	-1	
$A_{2u}$	1	1	-1	-1	-1	1	$z$
$E_u$	2	-1	0	-2	1	0	$(x, y)$

$\mathcal{D}_{3h}$	$E$	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$	
$A_1'$	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_2'$	1	1	-1	1	1	-1	$R_z$
$E'$	2	-1	0	2	-1	0	$(x, y)$ $(x^2 - y^2, 2xy)$
$A_1''$	1	1	1	-1	-1	-1	
$A_2''$	1	1	-1	-1	-1	1	$z$
$E''$	2	-1	0	-2	1	0	$(R_x, R_y)$ $(xz, yz)$



$C_{4v}$	$E$	$2C_4$	$C_4^2$	$2\sigma_v$	$2\sigma_d$	
$A_1$	1	1	1	1	1	$z$ $x^2 + y^2; z^2$
$A_2$	1	1	1	-1	-1	$R_z$
$B_1$	1	-1	1	1	-1	$x^2 - y^2$
$B_2$	1	-1	1	-1	1	$xy$
$E$	2	0	-2	0	0	$(x,y)$ $(R_x, R_y)$ $(xz, yz)$

Note: The  $\sigma_v$  planes in  $C_{4v}$  coincide with the  $xz$  and  $yz$  planes.

$D_{4h}$	$E$	$2C_4$	$C_4^2$	$2C_2$	$2C_2'$	$i$	$2S_4$	$\sigma_h$	$2\sigma_v$	$2\sigma_d$	
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_{2g}$	1	1	1	-1	-1	1	1	1	-1	-1	$R_z$
$B_{1g}$	1	-1	1	1	-1	1	-1	1	1	-1	$x^2 - y^2$
$B_{2g}$	1	-1	1	-1	1	1	-1	1	-1	1	$xy$
$E_g$	2	0	-2	0	0	2	0	-2	0	0	$(R_x, R_y)$ $(xz, yz)$
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1	
$A_{2u}$	1	1	1	-1	-1	-1	-1	-1	1	1	$z$
$B_{1u}$	1	-1	1	1	-1	-1	1	-1	-1	1	
$B_{2u}$	1	-1	1	-1	1	-1	1	-1	1	-1	
$E_u$	2	0	-2	0	0	-2	0	2	0	0	$(x,y)$

Note: The  $C_2$  axes in  $D_{4h}$  coincide with the  $x$  and  $y$  axes, and the  $\sigma_v$  planes with the  $xz$  and  $yz$  planes.

Note that the quantities  $\eta_{\pm} \equiv \frac{1}{2}(\sqrt{5} \pm 1)$  satisfy  $\eta_{\pm}^2 = 1 \pm \eta_{\pm}$  and  $\eta_+ \eta_- = 1$ .

$C_{5v}$	$E$	$2C_5$	$2C_5^2$	$5\sigma_v$	$\eta_{\pm} = \frac{1}{2}(\sqrt{5} \pm 1)$
$A_1$	1	1	1	1	$z$ $x^2 + y^2; z^2$
$A_2$	1	1	1	-1	$R_z$
$E_1$	2	$\eta_-$	$-\eta_+$	0	$(x,y)$ $(R_x, R_y)$ $(xz, yz)$
$E_2$	2	$-\eta_+$	$\eta_-$	0	$(x^2 - y^2, 2xy)$

$D_5$	$E$	$2C_5$	$2C_5^2$	$5C_2$	$\eta_{\pm} = \frac{1}{2}(\sqrt{5} \pm 1)$
$A_1$	1	1	1	1	$x^2 + y^2; z^2$
$A_2$	1	1	1	-1	$z$ $R_z$
$E_1$	2	$\eta_-$	$-\eta_+$	0	$(x,y)$ $(R_x, R_y)$ $(xz, yz)$
$E_2$	2	$-\eta_+$	$\eta_-$	0	$(x^2 - y^2, 2xy)$

$D_{5d}$	$E$	$2C_5$	$2C_5^2$	$5C_2$	$i$	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$	$\eta_{\pm} = \frac{1}{2}(\sqrt{5} \pm 1)$
$A_{1g}$	1	1	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_{2g}$	1	1	1	-1	1	1	1	-1	$R_z$
$E_{1g}$	2	$\eta_-$	$-\eta_+$	0	2	$\eta_-$	$-\eta_+$	0	$(R_x, R_y)$ $(xz, yz)$
$E_{2g}$	2	$-\eta_+$	$\eta_-$	0	2	$-\eta_+$	$\eta_-$	0	$(x^2 - y^2, 2xy)$
$A_{1u}$	1	1	1	1	-1	-1	-1	-1	
$A_{2u}$	1	1	1	-1	-1	-1	-1	1	$z$
$E_{1u}$	2	$\eta_-$	$-\eta_+$	0	-2	$-\eta_-$	$\eta_+$	0	$(x,y)$
$E_{2u}$	2	$-\eta_+$	$\eta_-$	0	-2	$\eta_+$	$-\eta_-$	0	

$\mathcal{D}_{5h}$	$E$	$2C_5$	$2C_5^2$	$5C_2$	$\sigma_h$	$2S_5$	$2S_5^3$	$5\sigma_v$	$\eta_{\pm} = \frac{1}{2}(\sqrt{5} \pm 1)$
$A_1'$	1	1	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_2'$	1	1	1	-1	1	1	1	-1	$R_z$
$E_1'$	2	$\eta_-$	$-\eta_+$	0	2	$\eta_-$	$-\eta_+$	0	$(x, y)$
$E_2'$	2	$-\eta_+$	$\eta_-$	0	2	$-\eta_+$	$\eta_-$	0	$(x^2 - y^2, 2xy)$
$A_1''$	1	1	1	1	-1	-1	-1	-1	$z$
$A_2''$	1	1	1	-1	-1	-1	-1	1	$(R_x, R_y)$
$E_1''$	2	$\eta_-$	$-\eta_+$	0	-2	$-\eta_-$	$\eta_+$	0	$(xz, yz)$
$E_2''$	2	$-\eta_+$	$\eta_-$	0	-2	$\eta_+$	$-\eta_-$	0	$(xz, yz)$

$C_{6v}$	$E$	$2C_6$	$2C_6^2$	$C_6^3$	$3\sigma_v$	$3\sigma_d$	
$A_1$	1	1	1	1	1	1	$z$
$A_2$	1	1	1	1	-1	-1	$R_z$
$B_1$	1	-1	1	-1	1	-1	
$B_2$	1	-1	1	-1	-1	1	
$E_1$	2	1	-1	-2	0	0	$(x, y)$
$E_2$	2	-1	-1	2	0	0	$(R_x, R_y)$
							$(xz, yz)$
							$(x^2 - y^2, 2xy)$

$\mathcal{D}_6$	$E$	$2C_6$	$2C_6^2$	$C_6^3$	$3C_2$	$3C_2'$	
$A_1$	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_2$	1	1	1	1	-1	-1	$z$
$B_1$	1	-1	1	-1	1	-1	$R_z$
$B_2$	1	-1	1	-1	-1	1	
$E_1$	2	1	-1	-2	0	0	$(x, y)$
$E_2$	2	-1	-1	2	0	0	$(R_x, R_y)$
							$(xz, yz)$
							$(x^2 - y^2, 2xy)$

$\mathcal{D}_{6h}$	$E$	$2C_6$	$2C_6^2$	$C_6^3$	$3C_2$	$3C_2'$	$i$	$2S_3$	$2S_6$	$\sigma_h$	$3\sigma_d$	$3\sigma_v$	
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2; z^2$
$A_{2g}$	1	1	1	1	-1	-1	1	1	1	1	-1	-1	$R_z$
$B_{1g}$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	
$B_{2g}$	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	
$E_{1g}$	2	1	-1	-2	0	0	2	1	-1	-2	0	0	$(R_x, R_y)$
$E_{2g}$	2	-1	-1	2	0	0	2	-1	-1	2	0	0	$(xz, yz)$
													$(x^2 - y^2, 2xy)$
$A_{1u}$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$z$
$A_{2u}$	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	
$B_{1u}$	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	
$B_{2u}$	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	
$E_{1u}$	2	1	-1	-2	0	0	-2	-1	1	2	0	0	$(x, y)$
$E_{2u}$	2	-1	-1	2	0	0	-2	1	1	-2	0	0	

# Cubic

$T$	$E$	$4C_3$	$4C_3^2$	$3C_2$	$\omega = \exp(2\pi i/3)$		
$A_1$	1	1	1	1	$x^2 + y^2 + z^2$		
$E$ {	1	$\omega$	$\omega^2$	1	$z^2 + \omega^2 x^2 + \omega y^2$		
	1	$\omega^2$	$\omega$	1	$z^2 + \omega x^2 + \omega^2 y^2$		
$T_2$	3	0	0	-1	$(x, y, z)$	$(R_x, R_y, R_z)$	$(yz, xz, xy)$

$T_d$	$E$	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
$A_1$	1	1	1	1	1	$x^2 + y^2 + z^2$	
$A_2$	1	1	1	-1	-1		
$E$	2	-1	2	0	0	$((2z^2 - x^2 - y^2), \sqrt{3}(x^2 - y^2))$	
$T_1$	3	0	-1	1	-1	$(R_x, R_y, R_z)$	
$T_2$	3	0	-1	-1	1	$(x, y, z)$	$(yz, xz, xy)$

$O$	$E$	$8C_3$	$3C_4^2$	$6C_4$	$6C_2$		
$A_1$	1	1	1	1	1	$x^2 + y^2 + z^2$	
$A_2$	1	1	1	-1	-1		
$E$	2	-1	2	0	0	$((2z^2 - x^2 - y^2), \sqrt{3}(x^2 - y^2))$	
$T_1$	3	0	-1	1	-1	$(x, y, z)$	$(R_x, R_y, R_z)$
$T_2$	3	0	-1	-1	1	$(xz, xy, yz)$	

$O_h$	$E$	$8C_3$	$3C_4^2$	$6C_4$	$6C_2$	$i$	$8S_6$	$3\sigma_h$	$6S_4$	$6\sigma_d$		
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$	
$A_{2g}$	1	1	1	-1	-1	1	1	1	-1	-1		
$E_g$	2	-1	2	0	0	2	-1	2	0	0	$((2z^2 - x^2 - y^2), \sqrt{3}(x^2 - y^2))$	
$T_{1g}$	3	0	-1	1	-1	3	0	-1	1	-1	$(R_x, R_y, R_z)$	
$T_{2g}$	3	0	-1	-1	1	3	0	-1	-1	1	$(xz, xy, yz)$	
$A_{1u}$	1	1	1	1	1	-1	-1	-1	-1	-1		
$A_{2u}$	1	1	1	-1	-1	-1	-1	-1	1	1		
$E_u$	2	-1	2	0	0	-2	1	-2	0	0		
$T_{1u}$	3	0	-1	1	-1	-3	0	1	-1	1	$(x, y, z)$	
$T_{2u}$	3	0	-1	-1	1	-3	0	1	1	-1		

# Icosahedral

$I_h$	$E$	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	$i$	$12S_{10}^3$	$12S_{10}$	$20S_6$	$15\sigma$	$\eta_{\pm} = \frac{1}{2}(\sqrt{5} \pm 1)$	
$A_g$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$	
$T_{1g}$	3	$\eta_+$	$-\eta_-$	0	-1	3	$\eta_+$	$-\eta_-$	0	-1	$(R_x, R_y, R_z)$	
$T_{2g}$	3	$-\eta_-$	$\eta_+$	0	-1	3	$-\eta_-$	$\eta_+$	0	-1		
$G_g$	4	-1	-1	1	0	4	-1	-1	1	0		
$H_g$	5	0	0	-1	1	5	0	0	-1	1	$(\sqrt{\frac{1}{12}}(2z^2 - x^2 - y^2), \frac{1}{2}(x^2 - y^2), xz, xy, yz)$	
$A_u$	1	1	1	1	1	-1	-1	-1	-1	-1		
$T_{1u}$	3	$\eta_+$	$-\eta_-$	0	-1	-3	$-\eta_+$	$\eta_-$	0	1	$(x, y, z)$	
$T_{2u}$	3	$-\eta_-$	$\eta_+$	0	-1	-3	$\eta_-$	$-\eta_+$	0	1		
$G_u$	4	-1	-1	1	0	-4	1	1	-1	0		
$H_u$	5	0	0	-1	1	-5	0	0	1	-1		

$C_{\infty v}$	$E$	$2C^z(\alpha)$	$\dots$	$\infty\sigma_v$		
$\Sigma^+ (A_1)$	1	1	$\dots$	1	$z$	$x^2 + y^2; z^2$
$\Sigma^- (A_2)$	1	1	$\dots$	-1	$R_z$	
$\Pi (E_1)$	2	$2\cos\alpha$	$\dots$	0	$(x, y)$	$(xz, yz)$
$\Delta (E_2)$	2	$2\cos 2\alpha$	$\dots$	0		$(x^2 - y^2, 2xy)$
$\Phi (E_3)$	2	$2\cos 3\alpha$	$\dots$	0		
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$		

$\mathcal{D}_{\infty h}$	$E$	$2C^z(\alpha)$	$\dots$	$\infty\sigma_v$	$i$	$2S^z(\alpha)$	$\dots$	$\infty C_2$	
$\Sigma_g^+ (A_{1g})$	1	1	$\dots$	1	1	1	$\dots$	1	$x^2 + y^2; z^2$
$\Sigma_g^- (A_{2g})$	1	1	$\dots$	-1	1	1	$\dots$	-1	$R_z$
$\Pi_g (E_{1g})$	2	$2\cos\alpha$	$\dots$	0	2	$-2\cos\alpha$	$\dots$	0	$(R_x, R_y)$
$\Delta_g (E_{2g})$	2	$2\cos 2\alpha$	$\dots$	0	2	$2\cos 2\alpha$	$\dots$	0	$(xz, yz)$
$\Phi_g (E_{3g})$	2	$2\cos 3\alpha$	$\dots$	0	2	$-2\cos 3\alpha$	$\dots$	0	$(x^2 - y^2, 2xy)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	
$\Sigma_u^+ (A_{1u})$	1	1	$\dots$	1	-1	-1	$\dots$	-1	$z$
$\Sigma_u^- (A_{2u})$	1	1	$\dots$	-1	-1	-1	$\dots$	1	
$\Pi_u (E_{1u})$	2	$2\cos\alpha$	$\dots$	0	-2	$2\cos\alpha$	$\dots$	0	$(x, y)$
$\Delta_u (E_{2u})$	2	$2\cos 2\alpha$	$\dots$	0	-2	$-2\cos 2\alpha$	$\dots$	0	
$\Phi_u (E_{3u})$	2	$2\cos 3\alpha$	$\dots$	0	-2	$2\cos 3\alpha$	$\dots$	0	
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	

### Selected tables for descent in symmetry

$C_{2v}$	$C_2$	$C_s$	$C_s$
		$(E, \sigma^{xz})$	$(E, \sigma^{yz})$
$A_1$	$A$	$A'$	$A'$
$A_2$	$A$	$A''$	$A''$
$B_1$	$B$	$A'$	$A''$
$B_2$	$B$	$A''$	$A'$

$\mathcal{D}_{3h}$	$C_{3v}$	$C_{2v}$	$C_s$	$C_s$
		$(\sigma_h \rightarrow \sigma^{yz})$	$(E, \sigma_h)$	$(E, \sigma_v)$
$A'_1$	$A_1$	$A_1$	$A'$	$A'$
$A'_2$	$A_2$	$B_2$	$A'$	$A''$
$E'$	$E$	$A_1 \oplus B_2$	$2A'$	$A' \oplus A''$
$A''_1$	$A_2$	$A_2$	$A''$	$A''$
$A''_2$	$A_1$	$B_1$	$A''$	$A'$
$E''$	$E$	$A_2 \oplus B_1$	$2A''$	$A' \oplus A''$

$\mathcal{D}_{\infty h}$	$C_{2v}$
$(x, y, z) \rightarrow$	$(x, z, y)$
$\Sigma_g^+$	$A_1$
$\Sigma_g^-$	$B_1$
$\Pi_g$	$A_2 \oplus B_2$
$\Delta_g$	$A_1 \oplus B_1$
$\dots$	$\dots$
$\Sigma_u^+$	$B_2$
$\Sigma_u^-$	$A_2$
$\Pi_u$	$A_1 \oplus B_1$
$\Delta_u$	$A_2 \oplus B_2$
$\dots$	$\dots$

$O(3)$	$O_h$	$\mathcal{T}_d$
$S_g$	$A_{1g}$	$A_1$
$P_g$	$T_{1g}$	$T_1$
$D_g$	$E_g \oplus T_{2g}$	$E \oplus T_2$
$F_g$	$A_{2g} \oplus T_{1g} \oplus T_{2g}$	$A_2 \oplus T_1 \oplus T_2$
$G_g$	$A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g}$	$A_1 \oplus E \oplus T_1 \oplus T_2$
$\dots$	$\dots$	$\dots$
$S_u$	$A_{1u}$	$A_2$
$P_u$	$T_{1u}$	$T_2$
$D_u$	$E_u \oplus T_{2u}$	$E \oplus T_1$
$F_u$	$A_{2u} \oplus T_{1u} \oplus T_{2u}$	$A_1 \oplus T_2 \oplus T_1$
$G_u$	$A_{1u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$	$A_2 \oplus E \oplus T_2 \oplus T_1$
$\dots$	$\dots$	$\dots$

## Reduction of a representation

If  $\Gamma = a_1\Gamma^{(1)} \oplus a_2\Gamma^{(2)} \oplus \dots \oplus a_n\Gamma^{(n)}$ , then

$$a_k = \frac{1}{h} \sum_R \chi^{(k)}(R) \chi^*(R),$$

where  $\chi(R)$  is the character of the operation  $R$  in the representation  $\Gamma$ ,  $\chi^{(k)}(R)$  is the character of the operation  $R$  in the representation  $\Gamma^{(k)}$ , and  $h$  is the number of elements in the group.

## Projection operators

The projection operator for representation  $\Gamma^{(k)}$  is

$$\mathcal{P}^{(k)} = \frac{n_k}{h} \sum_R \chi^{(k)}(R) R.$$

The projected function  $\mathcal{P}^{(k)}f$  obtained by applying  $\mathcal{P}^{(k)}$  to any function  $f$  is either zero or a component of a basis for representation  $\Gamma^{(k)}$ .

## Direct Products

Generally,

$$\chi^{\Gamma \otimes \Gamma'}(R) = \chi^\Gamma(R) \chi^{\Gamma'}(R),$$

and if the resulting representation is reducible it can be reduced in the usual way. Alternatively the following rules can be applied.

Treat  $g/u$  and  $'/'$  symmetry separately. For groups with an inversion centre,

$$g \otimes g = u \otimes u = g \quad \text{and} \quad g \otimes u = u.$$

For groups with a horizontal plane  $\sigma_h$  but no inversion centre, single and double primes,  $'$  and  $''$ , are used to denote symmetry and antisymmetry with respect to  $\sigma_h$ . Then

$$' \otimes ' = '' \otimes '' = ' \quad \text{and} \quad ' \otimes '' = ''.$$

Direct products involving nondegenerate representations are easily worked out from the character table. The product of any  $A$  or  $B$  with any  $E$  is an  $E$ , and the product of any  $A$  or  $B$  with any  $T$  is a  $T$ .

In the cubic groups  $\mathcal{T}_d$ ,  $O$  and  $O_h$ ,

$$E \otimes E = A_1 \oplus A_2 \oplus E,$$

$$E \otimes T_1 = E \otimes T_2 = T_1 \oplus T_2,$$

$$T_1 \otimes T_1 = T_2 \otimes T_2 = A_1 \oplus E \oplus T_1 \oplus T_2,$$

$$T_1 \otimes T_2 = A_2 \oplus E \oplus T_1 \oplus T_2.$$

For products of  $E_i$  with  $E_j$  in the axial groups the rules are complicated. If there is only one  $E$  representation, apart from  $g/u$  or  $'/'$  labels, it should be considered as  $E_1$ . We need the order  $n$  of the principal axis, which is usually obvious — e.g.  $n = 5$  for  $\mathcal{D}_{5h}$  — but for  $\mathcal{D}_{md}$  with  $m$  even,  $n = 2m$  (because  $\mathcal{D}_{md}$  has an  $S_{2m}$  axis when  $m$  is even).

(a) For  $E_i \otimes E_i$ :

(i) If  $E_{2i}$  exists, then

$$E_i \otimes E_i = A_1 \oplus A_2 \oplus E_{2i}.$$

(ii) Otherwise, if  $4i = n$  then

$$E_i \otimes E_i = A_1 \oplus A_2 \oplus B_1 \oplus B_2$$

(iii) Otherwise

$$E_i \otimes E_i = A_1 \oplus A_2 \oplus E_{|2i-n|}.$$

(b) For  $E_i \otimes E_j$  with  $i \neq j$ :

(i) If  $E_{i+j}$  exists, then

$$E_i \otimes E_j = E_{|i-j|} \oplus E_{i+j}.$$

(ii) If  $2(i+j) = n$ , then

$$E_i \otimes E_j = E_{|i-j|} \oplus B_1 \oplus B_2$$

(iii) Otherwise

$$E_i \otimes E_j = E_{|i-j|} \oplus E_{|i+j-n|}.$$

If there is only one  $A$  representation, apart from  $g/u$  or  $'/'$  labels, read  $A_1$  and  $A_2$  above as  $A$ ; similarly for  $B$ .

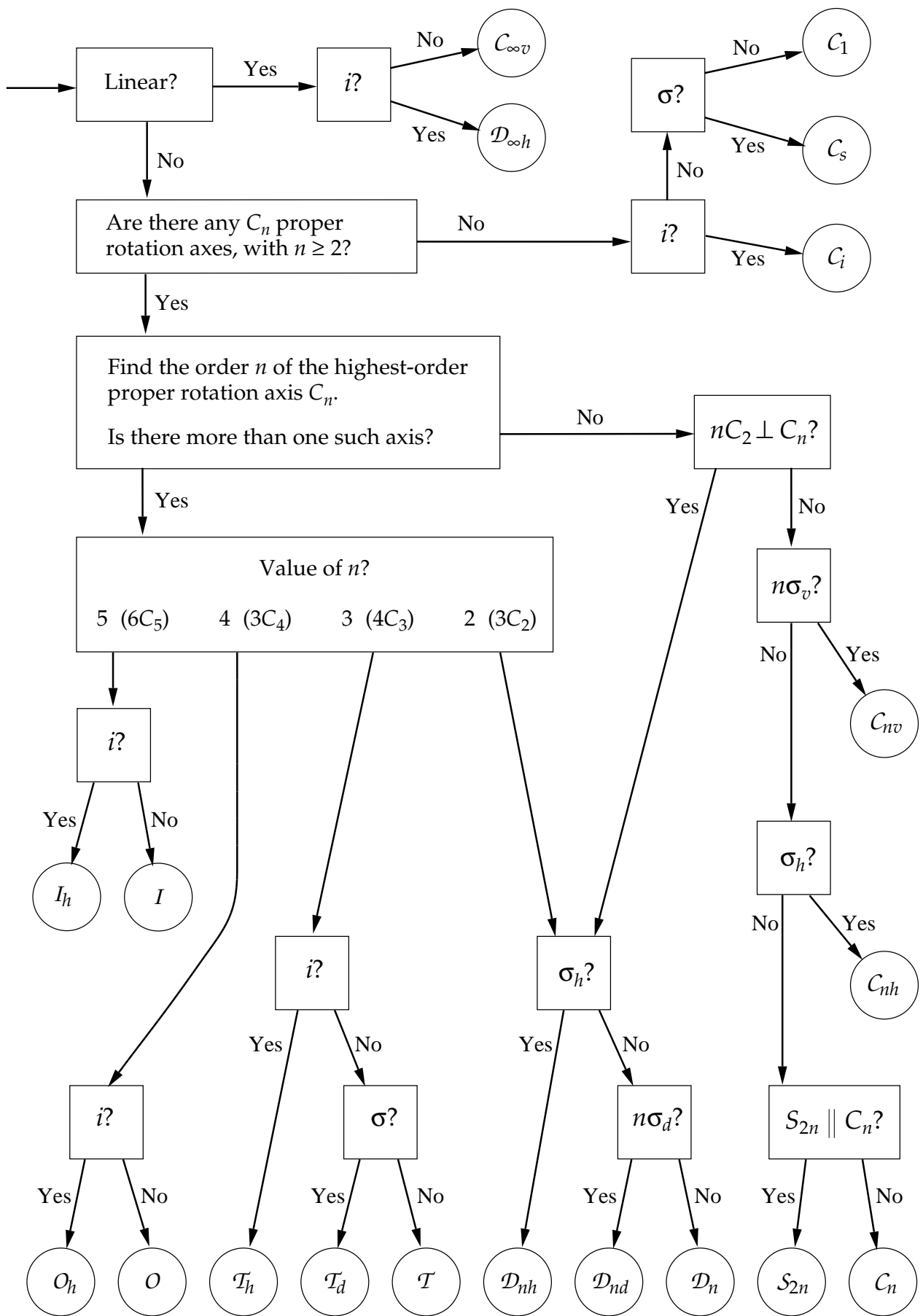
## Examples

For  $E \otimes E$  in  $C_{4v}$ : there is only one  $E$  representation, so treat it as  $E_1$ . Rule  $a(i)$  doesn't apply, because  $E_2$  doesn't exist, but  $a(ii)$  applies, so  $E \otimes E = A_1 \oplus A_2 \oplus B_1 \oplus B_2$ .

For  $E_{1g} \otimes E_{2u}$  in  $\mathcal{D}_{5d}$ , note first that  $g \otimes u = u$ . Then we need  $E_1 \otimes E_2$ , for which rule  $b(iii)$  applies, with  $n = 5$ , so the result is  $E_{1g} \otimes E_{2u} = E_{1u} \oplus E_{2u}$ .

## Antisymmetrized Squares

The antisymmetric component of  $E \otimes E$  or  $E_i \otimes E_i$  is always  $A_2$ . In the cubic groups, the antisymmetric part of  $T_1 \otimes T_1$  and  $T_2 \otimes T_2$  is  $T_1$ .



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## Space Groups

### General Equivalent Positions (GEPs) and Special Equivalent Positions (SEPs)

#### Space group $P2_1$

GEPs:

$$2 @ (x_n, y_n, z_n), (-x_n, \frac{1}{2} + y_n, -z_n)$$

SEPs:

None

#### Space group $P2_1/c$

GEPs:

$$4 @ (x_n, y_n, z_n), (-x_n, \frac{1}{2} + y_n, \frac{1}{2} - z_n), (x_n, \frac{1}{2} - y_n, \frac{1}{2} + z_n), (-x_n, -y_n, -z_n)$$

SEPs:

4 pairs:

$$2 @ (0, 0, 0) \text{ and } (0, \frac{1}{2}, \frac{1}{2})$$

$$2 @ (0, 0, \frac{1}{2}) \text{ and } (0, \frac{1}{2}, 0)$$

$$2 @ (\frac{1}{2}, 0, 0) \text{ and } (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$$

$$2 @ (\frac{1}{2}, \frac{1}{2}, 0) \text{ and } (\frac{1}{2}, 0, \frac{1}{2})$$

#### Space group $P2_12_12_1$

GEPs:

$$4 @ (x_n, y_n, z_n), (-x_n, \frac{1}{2} + y_n, \frac{1}{2} - z_n), (\frac{1}{2} + x_n, \frac{1}{2} - y_n, -z_n), (\frac{1}{2} - x_n, -y_n, \frac{1}{2} + z_n)$$

SEPs:

None

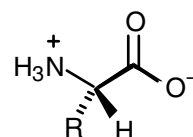
*Parameters for selected magnetic nuclei\**

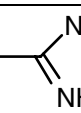

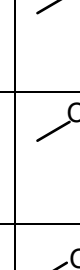
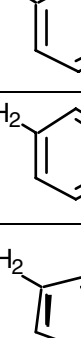
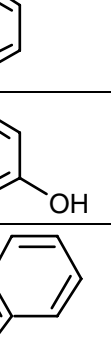
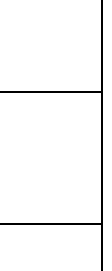
isotope	natural abundance (%)	spin, I
$^1\text{H}$	100	$\frac{1}{2}$
$^2\text{H}$	$1.5 \times 10^{-2}$	1
$^3\text{H}$	0	$\frac{1}{2}$
$^6\text{Li}$	7	1
$^7\text{Li}$	93	$\frac{3}{2}$
$^{10}\text{B}$	20	3
$^{11}\text{B}$	80	$\frac{3}{2}$
$^{13}\text{C}$	1	$\frac{1}{2}$
$^{14}\text{N}$	100	1
$^{15}\text{N}$	0.4	$\frac{1}{2}$
$^{17}\text{O}$	$3.7 \times 10^{-2}$	$\frac{5}{2}$
$^{19}\text{F}$	100	$\frac{1}{2}$
$^{23}\text{Na}$	100	$\frac{3}{2}$
$^{27}\text{Al}$	100	$\frac{5}{2}$
$^{29}\text{Si}$	5	$\frac{1}{2}$
$^{31}\text{P}$	100	$\frac{1}{2}$
$^{51}\text{V}$	100	$\frac{7}{2}$
$^{57}\text{Fe}$	2	$\frac{1}{2}$
$^{77}\text{Se}$	8	$\frac{1}{2}$
$^{103}\text{Rh}$	100	$\frac{1}{2}$
$^{107}\text{Ag}$	52	$\frac{1}{2}$
$^{109}\text{Ag}$	48	$\frac{1}{2}$
$^{113}\text{Cd}$	12	$\frac{1}{2}$
$^{119}\text{Sn}$	9	$\frac{1}{2}$
$^{129}\text{Xe}$	26	$\frac{1}{2}$
$^{195}\text{Pt}$	34	$\frac{1}{2}$
$^{203}\text{Tl}$	30	$\frac{1}{2}$
$^{205}\text{Tl}$	70	$\frac{1}{2}$
$^{207}\text{Pb}$	23	$\frac{1}{2}$

\*The list is not exhaustive.



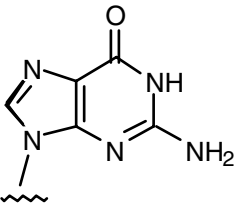
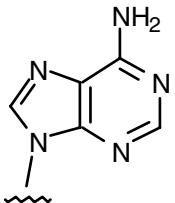
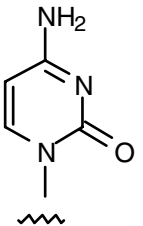
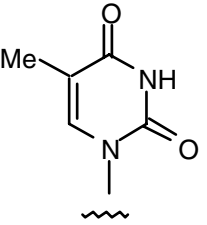
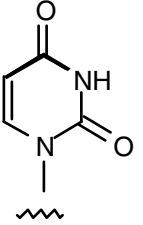
## Amino acids



Name	Three-letter code	Single-letter code	Side chain, R =
Serine	Ser	S	—CH <sub>2</sub> OH
Threonine	Thr	T	—CH(CH <sub>3</sub> )OH
Cysteine	Cys	C	—CH <sub>2</sub> SH
Methionine	Met	M	—CH <sub>2</sub> CH <sub>2</sub> SMe
Aspartic acid	Asp	D	—CH <sub>2</sub> COO <sup>-</sup>
Asparagine	Asn	N	—CH <sub>2</sub> CONH <sub>2</sub>
Glutamic acid	Glu	E	—CH <sub>2</sub> CH <sub>2</sub> COO <sup>-</sup>
Glutamine	Gln	Q	—CH <sub>2</sub> CH <sub>2</sub> CONH <sub>2</sub>
Lysine	Lys	K	—CH <sub>2</sub> CH <sub>2</sub> CH <sub>2</sub> CH <sub>2</sub> NH <sub>3</sub> <sup>+</sup>
Arginine	Arg	R	—CH <sub>2</sub> CH <sub>2</sub> CH <sub>2</sub> NH— 
Glycine	Gly	G	—H
Alanine	Ala	A	—Me
Leucine	Leu	L	—CH <sub>2</sub> CHMe <sub>2</sub>
Isoleucine	Ile	I	—CH(Me)CH <sub>2</sub> Me
Valine	Val	V	—CHMe <sub>2</sub>
Histidine	His	H	
Phenylalanine	Phe	F	
Tyrosine	Tyr	Y	
Tryptophan	Trp	W	
Proline*	Pro	P	

\*For proline the complete structure of the amino acid is shown.

### *Nucleotide bases*

Name	Abbreviation	Structure
Guanine	G	 <p>The structure of Guanine is a purine base consisting of a fused imidazole and pyrimidine ring system. It features a carbonyl group at the 6-position, an amino group at the 2-position, and a wavy line at the 9-position representing the attachment point to the sugar-phosphate backbone.</p>
Adenine	A	 <p>The structure of Adenine is a purine base consisting of a fused imidazole and pyrimidine ring system. It features an amino group at the 6-position and a wavy line at the 9-position representing the attachment point to the sugar-phosphate backbone.</p>
Cytosine	C	 <p>The structure of Cytosine is a pyrimidine base consisting of a single six-membered ring. It features an amino group at the 4-position, a carbonyl group at the 2-position, and a wavy line at the 1-position representing the attachment point to the sugar-phosphate backbone.</p>
Thymine	T	 <p>The structure of Thymine is a pyrimidine base consisting of a single six-membered ring. It features a methyl group (Me) at the 5-position, carbonyl groups at the 2 and 4 positions, and a wavy line at the 1-position representing the attachment point to the sugar-phosphate backbone.</p>
Uracil	U	 <p>The structure of Uracil is a pyrimidine base consisting of a single six-membered ring. It features carbonyl groups at the 2 and 4 positions and a wavy line at the 1-position representing the attachment point to the sugar-phosphate backbone.</p>